Weekday Dependence in Reliability Analysis of Repairable Systems

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Abstract

A useful and informative way of representing the reliability behavior of repairable systems is plotting the Mean Cumulative Function (MCF) and the Recurrence Rate (RR) versus the age or calendar date in days of the observed systems. Usually this approach implicitly assumes that systems operate uniformly 24 hours per day and seven days per week, the so-called 24/7. We analyzed weekday dependence statistics of failures and found that often frequency of failures on weekends was significantly lower than on work days. More detailed analysis showed that all systems could be separated into two groups: “5d systems” that never had failures on Saturdays and Sundays and “7d systems” that had failures any day of the week.
Typically, the ratio of average 24/7 weekly RR's for these two groups is close to 5:7. The implication is that "5d systems" do not age during week-ends. Neglecting this fact and aggregating statistics of systems of both types can lead to extra noise and bias in parameter estimation.

Key Words: Reliability analyses, field data, mean cumulative function, repairable system

Analysis of Aggregated Systems

Plotting the average number of failures across systems, called the Mean Cumulative Function (MCF), and its derivative, called the Recurrence Rate (RR), versus the age or calendar date in days of observed systems is a useful and informative way of representing the reliability behavior of repairable systems.
This approach is widely used in Sun Microsystems for analysis of statistics of failures both for hardware (datacenters, servers, storages, network routers, and so on) and software. This method is far more informative than using common single value metrics such as Mean Time between Failure (MTBF) and a Mean Time to Repair/Recovery (MTTR). These summary statistics typically assume a homogeneous Poisson process (HPP), that is, a renewal process where the times between events are derived from a single distribution, are independent, and exponentially distributed with a constant rate of occurrence. These presumptions are often not satisfied in practice, and so MCFs, a non-parametric approach, are increasingly being used to monitor the reliability of repairable systems in the field [2,3,4].

The reliability behaviour of a single machine can best be shown as a cumulative plot, which graphs the number of failures (outages) versus time, where time can be age from installation or a calendar date. The MCF, which shows the average number of failures across all systems versus time, represents the behavior of a group of machines.

There is an implicit assumption that systems operate uniformly 24 hours per day and seven days per week, the so-called 24/7. However, this assumption could be suspect for many important practical situations.
We might expect weekday dependence for several reasons:

1. Reduced work hours on weekends cause reduced loads on systems. This effect should not change the types of failure modes observed.

2. Maintenance is often done on week-ends and may include changing of software and hardware configurations, with initial application and test on Mondays of the new configurations. These procedures have the potential to introduce new failure modes different from those in normal operation.

To investigate possible effects we analyzed the frequency distributions of numbers of failures vs. weekday from a large database of systems in the field. Results are shown on Chart 1.

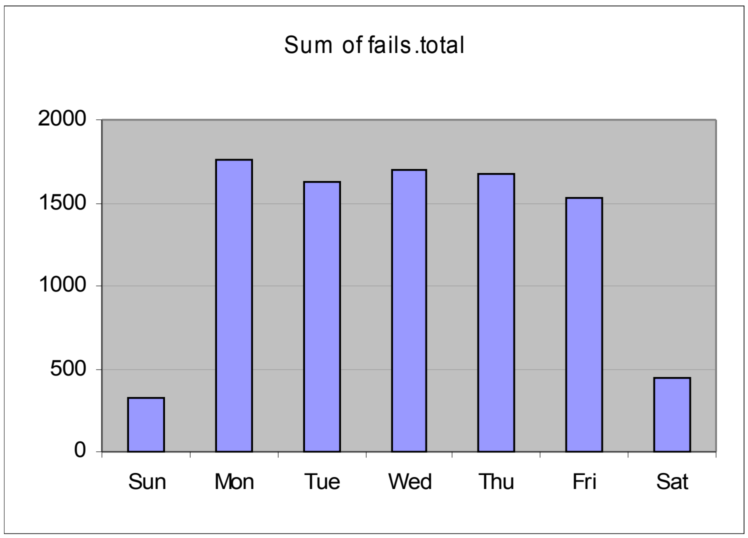

Chart 1: Number of failures vs. weekday

It is obvious from the Chart 1 that
a) The number of failures is much lower on weekends: about 25% of the mean workday level.
b) Considering just the workdays (Monday through Friday), the number of failures on Mondays is higher (106%) and on Fridays is lower (92%) than the five day workday average.
A □□- test for homogeneity shows that a null-hypothesis of constant frequency during M-F workdays is rejectable with a p-value of 0.0013.

It is interesting to compare the weekday dependence of the number of failures that is shown on Chart 1 with the weekday dependence of other characteristics such as the total number of active systems (Chart 2) and the number of systems installed on specific day (Chart 3).

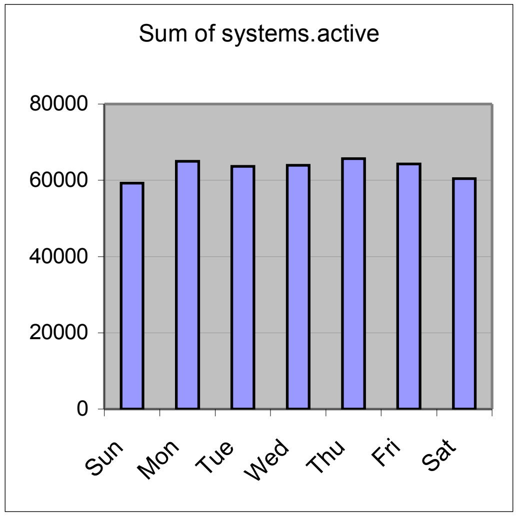

Chart 2: Total number of active systems vs. weekday

We see from Chart 2 that slightly fewer systems are active on weekends compared to weekdays (within +4.0%, -6.3% of the overall average), but this small difference does not account for the observed behavior.

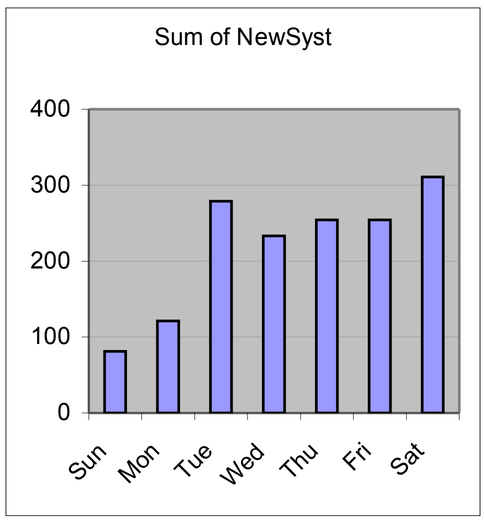

Chart 3: Number of new systems installed on specific weekday

Chart 3 shows that numbers of new systems installed are lower on Sundays and Mondays and higher on Tuesdays and Saturdays.

It is possible that different failure modes can show weekday dependence due to varying usage and maintenance patterns of systems. Chart 4 shows this comparison, which does not confirm this hypothesis.

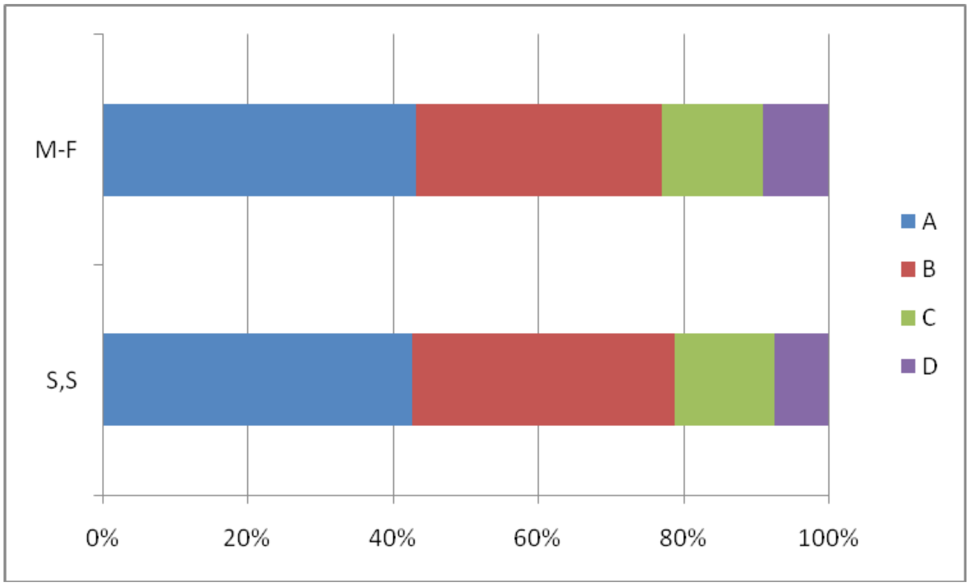

Chart 4: Proportions of sums of different failure modes

We see from the Chart 4 that there are no essential differences in proportions for the sums of the different types of failures comparing weekdays to weekends. The □□- test for homogeneity has a p-value of 0.52. The failure distributions are basically the same.

The weekday effect introduces extra noise (or adds a periodic component) to charts of RR and MCF. If we calculate RR from the MCF chart and we choose a time window that does not have a whole number multiple of weeks, then we may bias the results and add variation.

To illustrate, let us assume we have a 5d system that has 1 failure every weekday and no failures on weekends. We calculate a moving average RR using 3-day windows. Starting from a Sunday (that is, S-M-T), we get the following seven point sequence for the RR: 2/3, 1, 1, 1, 2/3, 1/3, 1/3. If we use only weekdays, we get 1, 1, 1, 1, 1. In the first case we have a bias and a periodic component that are absent in the second approach. However, if we use weekly averaging over a 7 day window, we get the following sequence for RR (starting from a Sunday): 5/7, 5/7, 5/7, 5/7, 5/7, 5/7, 5/7.

Chart 5 compares the recurrence rates estimated using 19 and 21 day windows. There are slight differences, but separating out the weekday effect from the smoothing effect of a larger window is not easily accomplished by looking at the RRs.

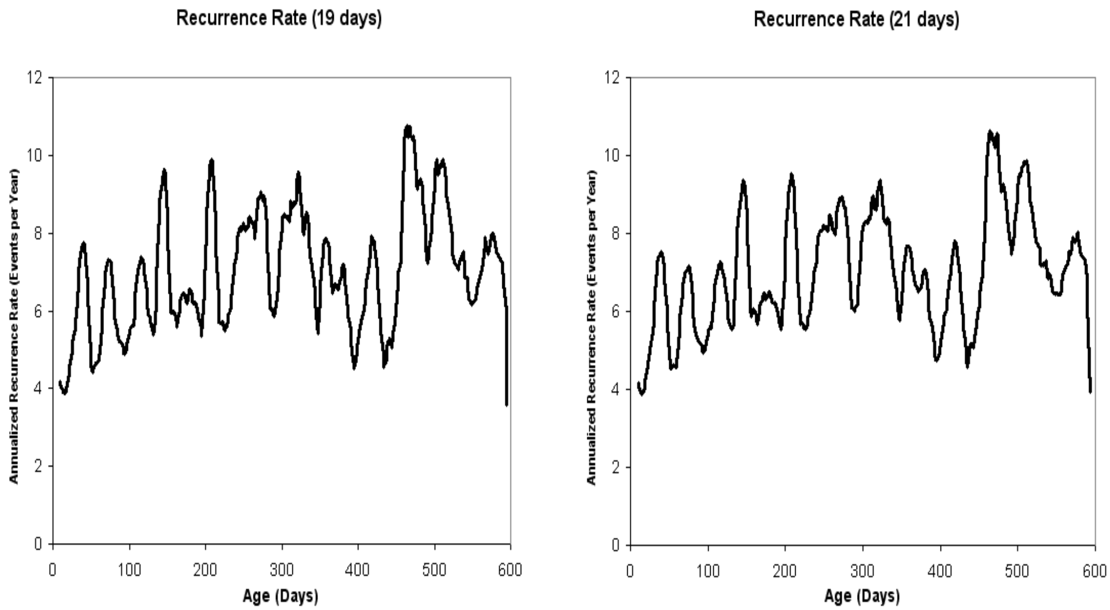

Chart 5: RR calculated with 19 and 21 day windows. It is difficult to separate noise from a periodic component, related to the weekday dependence.

To separate the week-day effect from age-effect we could use trend-period-noise decomposition in the same way as it is done for data with seasonality [6]:

RR(t) = Trend(t) * F(Weekday(t)) * Noise(t).

This method can be complicated. Alternatively,
we can use two simpler approaches to exclude the week-day effect:
1. Aggregate data to whole weeks and plot weekly instead of daily data. Then with or without an age effect we could expect the same frequency of failure every week.
2. Associate the frequency of failure on different weekdays as result of different number of working hours each day and plot on x-axis the cumulative number of work hours instead of days.

The first way is simpler and may be accurate enough, because after the aggregation we still can do trend - noise decomposition. As we see from Chart 1, "by week", we can obtain the trend clear of noise by choosing the spreadsheet functions TREND or AVERAGEA with the smoothing window at least 7 weeks. The trend line still contains the age effect but is clear of week-day effect and noise.

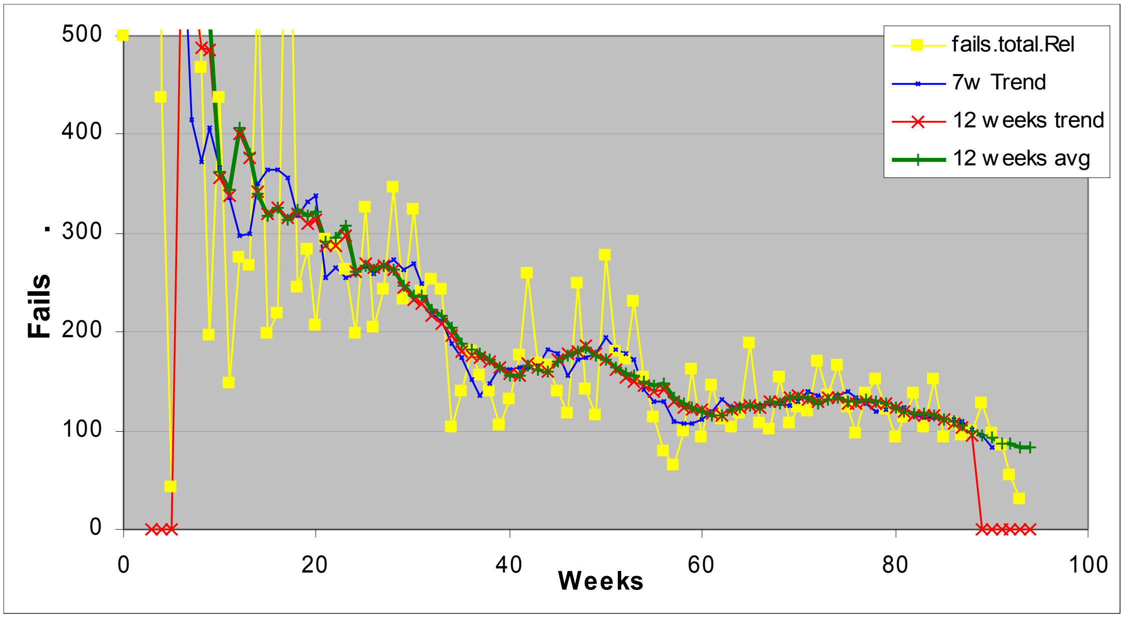

Chart 6: RR calculated with 7 and 12 weeks window.

By-System Analysis

It is interesting to analyze the weekday failure distribution by systems.
For each system we calculate a week-end index (WEI) and Monday and Friday indices:

WEI = (Number of Weekend failures*7/2) / Total # of failures

For 5d systems, WEI should be zero. For 7d systems, WEI should be near 1.

MonInd = (Number of Monday failures*5) / Total # of workday failures
FriInd = (Number of Friday failures*5) / Total # of workday failures

For no Monday or Friday effects, MonInd and FriInd should be near 1.

The data consisted of 934 systems with known serial numbers and installation dates. We do not analyze early failures here, and so we account only for failures that happened after one week. We split the systems into three groups:
"5 work day systems" ("5d") - 74% of the systems that never had a week-end failure
"7 work day systems" ("7d") - 22% of the systems that had week-end index >= .25
"Intermediate systems" ("i") - 4% of the systems that had week-end index between 0 and .25 .

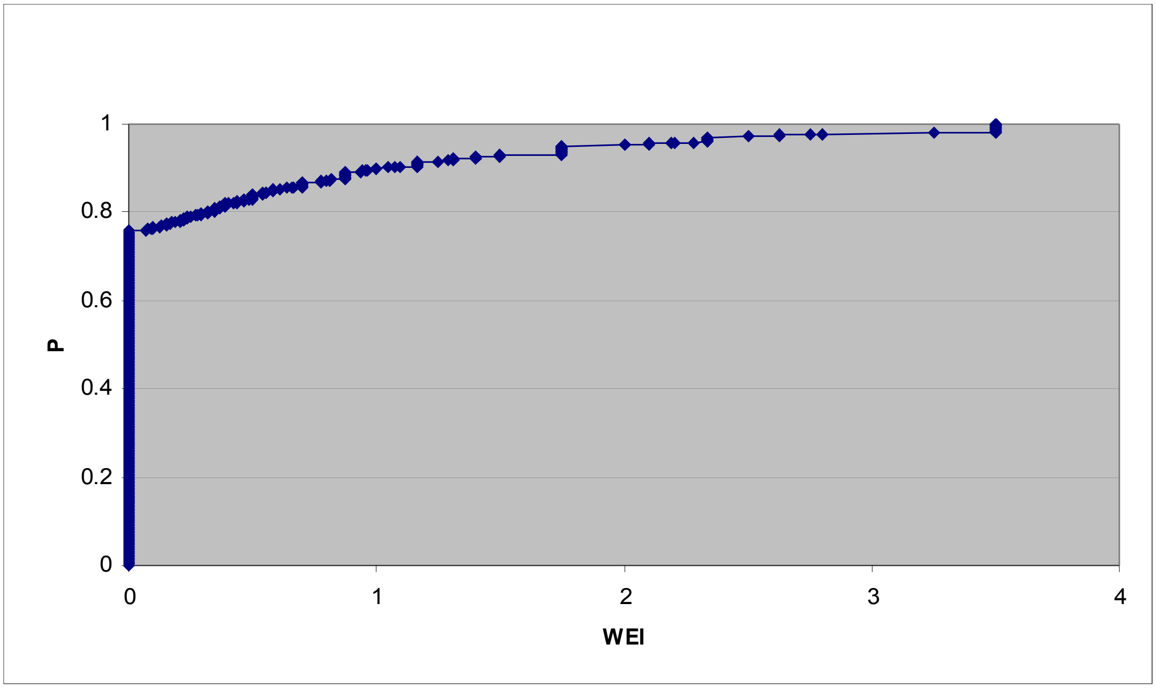

Chart 7: Distribution of the Weekend Index (WEI).

To validate the hypothesis that the absence of week-end failures is the result of reporting week-end failures on Monday, we calculate MonInd separately for 5d and 7d systems. If there is the same failure rate on week-ends as work days with misreporting, we would have MonInd for 5d systems about 3 times higher than for 7d systems. We found that in reality MonInd for 5d systems is only about 6% higher than for 7d systems, and so the misreporting does not play an essential role.

To validate the hypothesis that 5d systems do not age at the same rate during week-ends we compare MCFs for 5d and 7d systems.
The results are in Chart 8 and Chart 9.

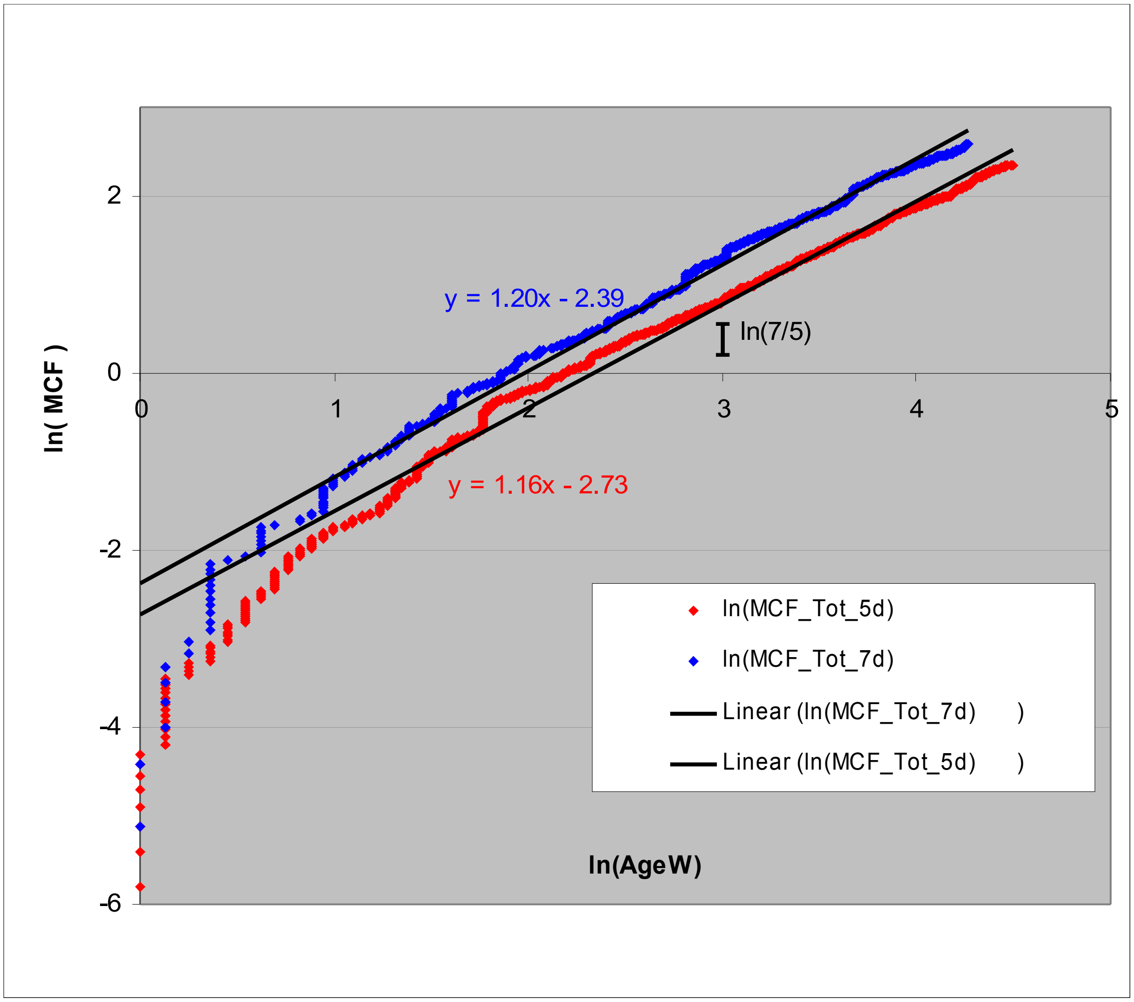

Chart 8: Duane plots for 5d and 7d systems, point=system.

We see that Duane plots for MCF have very close values of the slope β for 5d and 7d systems (trend lines are almost parallel) and the ratio of the intercepts in Chart 8
$\alpha\_7d / \alpha\_5d = \exp(-1.81)/\exp(-2.22) = 1.5$ is quite close to 7/5 = 1.4 . Thus, it looks as if 5d systems really do not age during week-ends.

The difference between MCF plots on Chart 8 and Chart 9 is the result of plotting 1 point = 1 failure on Chart 8 and 1 point = 1 week on Chart 9. The data consists of more systems with small ages and fewer systems with large ages. Consequently, the weight of small ages is higher on Chart 8. We would expect that if we had the constant number of systems for all ages, then the result should be closer to Chart 9. So we consider results 1 point = 1 week on Chart 9 to be a more accurate representation.

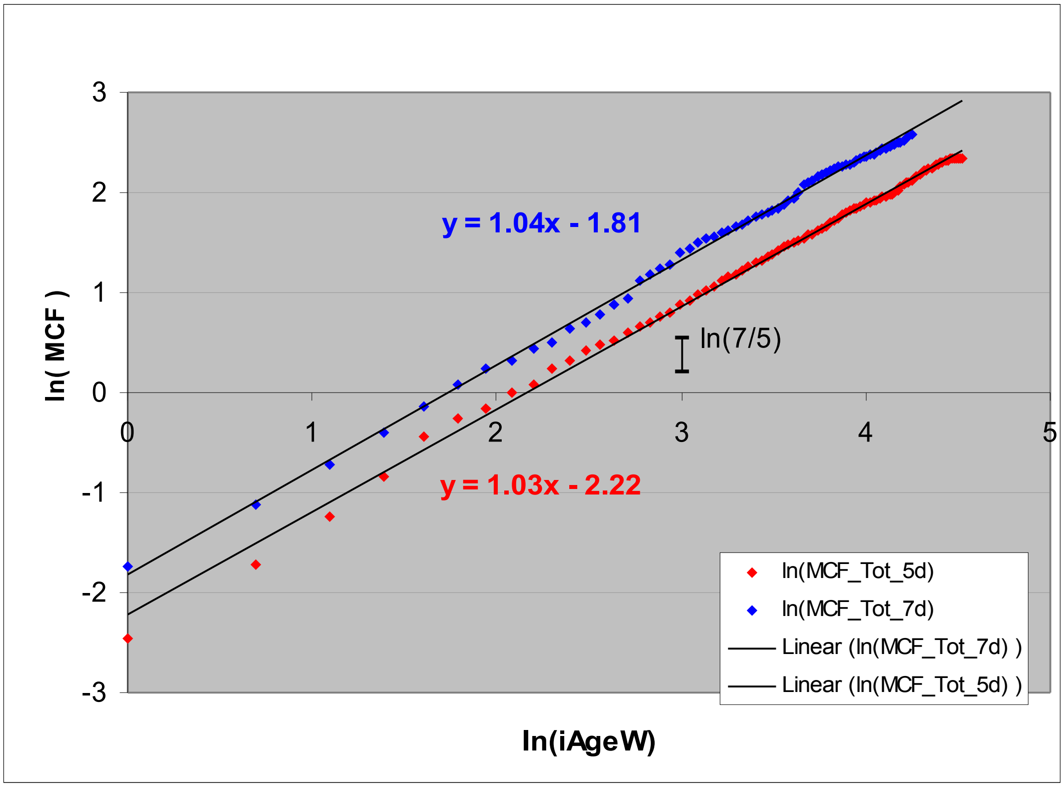

Chart 9: Duane plots for 5d and 7d systems, point=week.

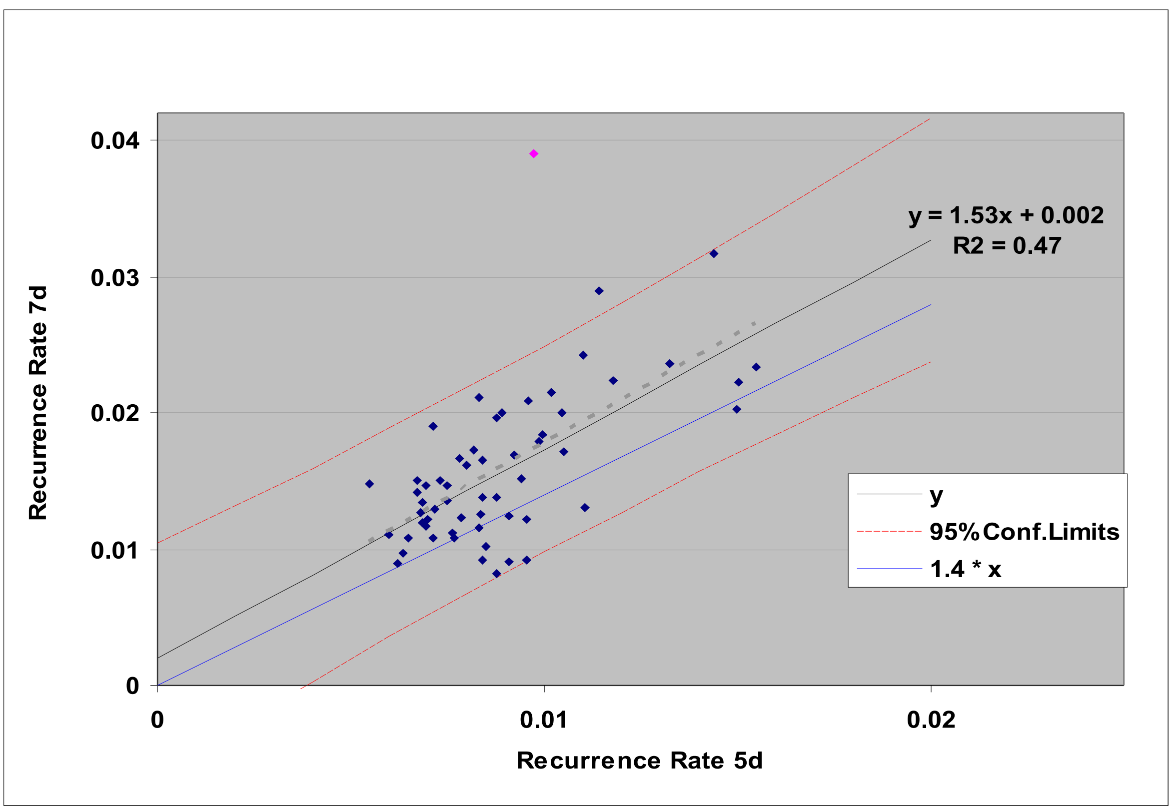

Chart 10: Comparing Recurrence Rates across customers

Chart 10 shows a comparison of average RRs for 5d and 7d systems across several customers that have 10 or more systems of each type. The average RR was obtained by dividing the total number of failures by the total age in days for each type of system..

One clearly sees that RR for 7d systems is usually significantly higher than RR for 5d systems, with a ratio that is about 1.6, which is even higher than 7/5 = 1.4. The obvious inference is that the 7d systems fail at a higher rate over 7 days than the 5d systems fail over 5 days.

Conclusions

We see that weekday dependence is significant in failure behavior and must be taken into account using work days instead of calendar days or analyzing statistics of failure for 5d and 7d systems separately.

Neglecting this fact and aggregating statistics of systems of both types can lead to extra noise and possible bias in parameter estimation.

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